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Candidates must write the Set No. on the title page of the answer book.

**SAHODAYA PRE BOARD EXAMINATION - 2023-24**

- Please check that this question paper contains **07** printed pages.
- Set number given on the right-hand side of the question paper should be written on the title page of the answer book by the candidates.
- Check that this question paper contains **38** questions.
- Write down the Serial Number of the question in the left side of the margin before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed 15 minutes prior to the commencement of the examination. The students will read the question paper only and will not write any answer on the answer script during this period. Students should not write anything in the question paper.

**CLASS- XII****SUB : MATHEMATICS (041)****Time Allowed: 3 Hours****Maximum Marks : 70****General Instructions :**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.



10. If  $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$  is such that  $A^2 = I$  then

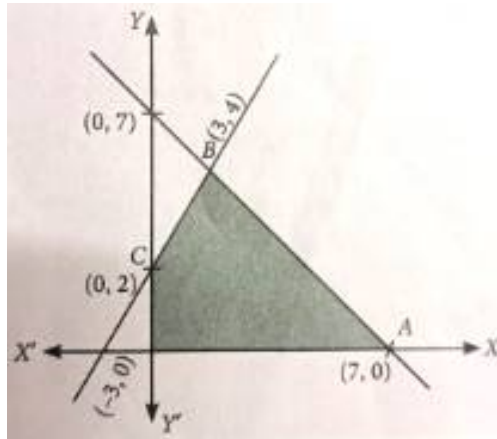
(a)  $1 + \alpha^2 + \beta\gamma = 0$

(b)  $1 - \alpha^2 + \beta = 0$

(c)  $1 - \alpha^2 - \beta\gamma = 0$

(d)  $1 + \alpha^2 - \beta\gamma = 0$

11. The shaded region in the figure represents the solution of which system of inequalities?



(a)  $x + y \geq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(b)  $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$

(c)  $x + y \leq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

(d)  $x + y \geq 7, 2x - 3y + 6 \leq 0, x \geq 0, y \geq 0$

12. If a line makes an angle  $\frac{\pi}{3}$  with each X and Y axes then the obtuse angle made with Z axis is

(a)  $\frac{3\pi}{2}$

(b)  $\frac{2\pi}{3}$

(c)  $\frac{3\pi}{4}$

(d)  $\frac{5\pi}{6}$

13. If A is a square matrix of order 3, such that  $A(\text{adj } A) = 10I$ , then  $|\text{adj } A|$  is equal to

(a) 1

(b) 10

(c) 100

(d) 101

14. A and B are events such that  $P(A/B) = P(B/A)$  then

(a)  $A \subset B$

(b)  $B = A$

(c)  $A \cap B = \emptyset$

(d)  $P(A) = P(B)$

15. Integrated factor of the differential equation  $x \frac{dy}{dx} - 2y = 4x^2$  is

(a)  $x^2$

(b)  $\frac{1}{x^2}$

(c)  $-\frac{1}{x^2}$

(d) none of these

16. The direction cosines of vector  $\overrightarrow{BA}$ , where coordinates of A and B are  $(1, 2, -1)$  and  $(3, 4, 0)$  respectively, are:

(a)  $-2, -2, -1$

(b)  $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

(c)  $2, 2, 1$

(d)  $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

17. If  $y = \sec x^0$ , then  $\frac{dy}{dx}$  is equal to:

- (a)  $\sec x \tan x$                       (b)  $\sec x^\circ \tan x^\circ$                       (c)  $\frac{\pi}{180} \sec x^\circ \tan x^\circ$                       (d) None of these

18. The number of lines passing through the origin which make equal angles with the coordinate axes is

- (a) 1                                      (b) 4                                      (c) 8                                      (d) 2

### ASSERTION-REASON BASED QUESTIONS

In the following two questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.  
(b) Both A and R are true but R is not the correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.

19. **Assertion (A):** Let  $A = \{-1, 1, 2, 3\}$  and  $B = \{1, 4, 9\}$ , where  $f : A \rightarrow B$  given by  $f(x) = x^2$ , then  $f$  is a many-one function.

**Reason (R):** If  $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ , for every  $x_1, x_2 \in \text{domain}$ , then  $f$  is one-one or else many-one.

20. **Assertion (A):** The two positive numbers  $x$  and  $y$  are such that  $x+y=35$  and  $x^2 y^5$  is maximum, then the numbers are 10 and 25.

**Reason(R):** If  $f$  be a function, defined on an interval  $I$  and  $c \in I$  and also, if  $f$  be twice differentiable at  $c$ , then  $x=c$  is a point of local maximum if  $f'(c) = 0$  and  $f''(c) < 0$  and the value of  $f(c)$  is local maximum value of  $f$ .

### SECTION B

**This section comprises of very short answer type-questions (VSA) of 2 marks each**

21. Find the domain of  $f(x) = \sin^{-1} \sqrt{x-1}$ .

**OR**

Find the value of  $\sin(2 \tan^{-1}(0.75))$ .

22. Find the least value of 'a' such that the function is given by  $f(x) = x^2 + ax + 1$  is strictly increasing on  $(1, 2)$ .

23. Find the maximum value of the function  $f(x) = \sin x + \cos x$ .

**OR**

A particle moves along the curve  $\frac{x^2}{16} + \frac{y^2}{4} = 1$ . Find the co-ordinates of the points, where the rate of change of abscissa is 4 times that of its ordinate.

24. Evaluate  $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx$

25. Divide 64 into two parts such that sum of the cubes of two parts is minimum.

### SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate :  $\int \frac{\cos x}{\sqrt{3-3 \sin x - \cos^2 x}} dx$

27. Two defective bulbs are accidentally mixed with 6 good ones. If three bulbs are drawn at random, find the mean of the number of defective bulbs drawn.

28. Evaluate:  $\int \left\{ \log (\log x) + \frac{1}{(\log x)^2} \right\} dx$

**OR**

Evaluate :  $\int_0^4 (|x| + |x - 2| + |x - 4|) dx$

29. Solve  $(1 + e^{x/y}) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

**OR**

Solve  $\frac{dy}{dx} = \sec (x + y)$

30. Solve the following linear programming problem Minimize  $Z = 3x + 5y$  subject to constraints.

$$x + 3y \geq 3, x + y \geq 2, x \geq 0, y \geq 0$$

**OR**

Solve the following linear programming problem Maximize  $Z = -x + 2y$  subject to constraints.

$$-x + 3y \leq 0, x + y \leq 6, x - y \leq 2, x \geq 0, y \geq 0$$

31. If  $x = a(2\theta - \sin 2\theta)$  and  $y = a(1 - \cos 2\theta)$ , find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{3}$ .

### SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Find the area bounded by the curve  $y^2 = 4a^2(x - 1)$  and the lines  $x = 1$  and  $y = 4a$ .

33. If  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ ,  $6A^{-1} = A^2 + cA + dI$  then find the value of  $(c, d)$ .

34. Prove that the relation  $R$  on the set  $N \times N$  defined by  $(a, b) R (c, d)$  iff  $ad(b + c) = bc(a + d)$  for all  $(a, b), (c, d) \in N \times N$  is an equivalence relation.

**OR**

Let  $f: R_+ \rightarrow [-5, \infty)$  be a function defined as  $f(x) = 9x^2 + 6x - 5$ . Show that  $f$  is bijective.

35. Find the image of the point  $(1, 6, 3)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ . Also, write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

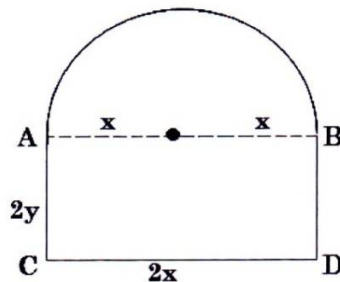
**OR**

Show that the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y-1}{2} = z$  intersect. Also find the equation of a line passing through the point of intersection of the above lines and parallel to  $z$ -axis.

### SECTION E

(This section comprises of 3 case-study/passage-based questions 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:** Read the following passage and answer the questions given below.



Mr Shashi, who is an architect, designs a building for a small company.

The design of window on the ground floor is proposed to be different from other floors.

The window is in the shape of a rectangle which is surmounted by a semi-circular opening.

This window is having a perimeter of 10 m.

- If  $2x$  and  $2y$  represents the length and breadth of the rectangular portion of the windows then, find the value of  $y$ , in terms of  $x$ .
- Express the combined area ( $A$ ) of the rectangular region and semi-circular region of the window, as a function of  $x$ .
- Find the maximum value of area  $A$  of the whole window using derivatives.

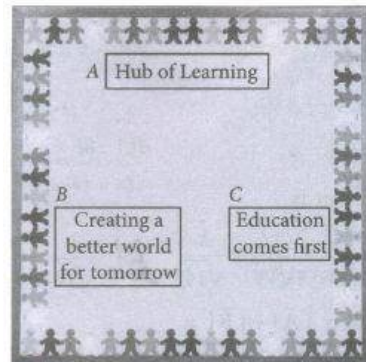
**OR**

- (iii) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, find the length and breadth of rectangular portion of the window. Also find the radius of semi-circular opening of the window. Use derivatives.

**37. Case-Study2::**

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.

Based on the above information, answer the following questions.



- (i) Let  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  be the position vectors of points A, B and C respectively, then find the value of  $\vec{a} + \vec{b} + \vec{c}$ .
- (ii) What will be the area of  $\Delta ABC$ ?
- (iii) Suppose, if the given slogans are to be placed on a straight line, then find the value of  $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

**OR**

If  $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$  then find the unit vector in the direction of vector  $\vec{a}$ .

38. **Case-Study 3:** Three friends A, B and C are given a situation by the company to find the correct solution. A's, B's, and C's chances of finding the correct solution are 1 out of 3, 2 out of 7 and 3 out of 8 respectively.



Basing on the above concept answer the following

- (i) Find the probability that exactly one of them will find a correct solution.
- (ii) Find the probability that problem is not solved, when all the three try.

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