DAV PUBLIC SCHOOLS , ODISHA SET-1

PRE-BOARD EXAMINATION
(2023-24)

- Please check that this question paper contains 6 printed Pages.
- Check that this question paper contains 38 questions.
- Write down the Serial Number of the question in the left side of the margin before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed 15 minutes prior to the commencement of the examination. The students will read the question paper only and will not write any answer on the answer script during this period.

CLASS- XII
SUB : MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks:80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is Compulsory. However, there are internal choices in some questions.
2. Section **A** has **18 MCQ's** and **02** Assertion-Reason based questions of **1** mark each.
3. Section **B** has **5Very Short Answer (VSA)-** type questions of **2** marks each.
4. Section **C** has **6 Short Answer (SA)-** type questions of **3** marks each.
5. Section **D** has **4 Long Answer (LA)-** type questions of **5** marks each.
6. Section **E** has **3** source based/case based/passage based/integrated units of assessment (**4** marks each) with sub parts.

SECTION-A

1. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

   (a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta = 0$ (c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

2. If A is a square matrix of order 3 such that det(3A) = 27, then det(A) is

   (a) 1 (b) 27 (c) 9 (d) 3

3. If A and B are square matrices of order 2 such that 2A + 3B is a null matrix such that det(A) = 9, then det(B) is

   (a) 6 (b) -4 (c) -6 (d) 4

4. If the function $f(x) = \begin{cases} 2x + 1, & x < 1 \\ ax^2 + b, & x \geq 1 \end{cases}$ is differentiable at x = 1. Then (a, b) is

   (a) (1, 1) (b) (1, 2) (c) (2, 1) (d) (- 1, 1)

5. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axes, is

   (a) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2}$ (b) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (c) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ (d) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

6. The product of order and degree of differential equation

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = k.\frac{d^2y}{dx^2} \text{ is}$$

(a)4 (b)5 (c)6 (d) can't be determined

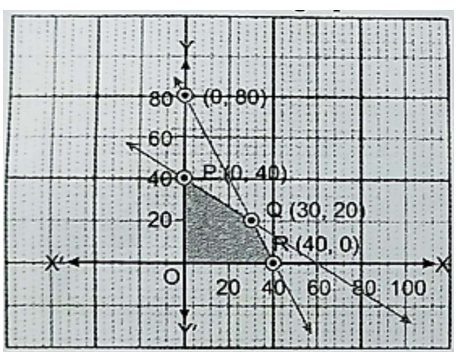
7. The constraints of a linear programming problem are $x + 2y \geq 3, x \geq 10, y \geq 0$
Which of the following objective function has an optimal solution with respect to above set of constraints?
(a) Minimize Z = x + y
(b) Minimize Z = 0.5 x + y
(c) Maximize Z = x + y
(d) Maximize Z = 2x + y

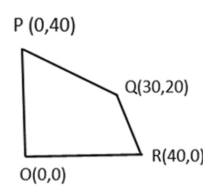
8. If $\vec{a}.\hat{\imath} = \vec{a}.(\hat{\imath}+\hat{\jmath}) = \vec{a}.(\hat{\imath}+\hat{\jmath}+\hat{k}) = 1$, then $\vec{a}$ is
(a) $\hat{k}$ (b) $\hat{\imath}$ (c) $\hat{\jmath}$ (d) $\hat{\imath}+\hat{\jmath}+\hat{k}$

9. The value of $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x}+\sqrt{21-x}} dx$ is
(a) $\frac{21}{2}$ (b) 0 (c) $\frac{5}{2}$ (d) $-\frac{21}{2}$

10. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then P is equal to
(a) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

11. For an LPP the objective function is Z=4x+3y and the feasible region determined by a set of constraints (linear inequations) is shown in the graph





Which of the following statements is true?
(a) Maximum value of Z is at R.
(b) Maximum value of Z is at Q
(c) Value of Z at R is less than the value at P.
(d) The value of Z at Q is less than the value at R.

12. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of vertices A, B, C of a parallelogram ABCD, then position vector of D is
(a) $\vec{a}-\vec{c}+\vec{b}$ (b) $\vec{a}+\vec{c}-\vec{b}$ (c) $\vec{a}-\vec{c}-\vec{b}$ (d) $\vec{c}-\vec{a}+\vec{b}$

13. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^{2021} is
(a) $\begin{bmatrix} 2021 & 2021 \\ 0 & 2021 \end{bmatrix}$ (b) $\begin{bmatrix} 2021 & 1 \\ 0 & 2021 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2021 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

14. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability of their making a common error is $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

(a) $\frac{1}{12}$ (b) $\frac{1}{40}$ (c) $\frac{10}{13}$ (d) $\frac{13}{120}$

15. The number of solution of $\frac{dy}{dx} = \frac{y+1}{x+1}$ when y(1)=2

(a)none (b) one (c) two (d) Infinite

16. If $\vec{a} \times \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $|\vec{a}| = 2, |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 30^0 (b) 60^0 (c) 90^0 (d)120^0

17. If $y = 5e^{7x} + 6e^{-7x}$ then y_2 is

(a)35y (b) 42y (c) 49y (d) none of these

18. If direction ratios of a line are $2,6,-3$ and it makes obtuse angle with y-axis, then its direction cosines are

(a) $\frac{2}{7}, \frac{6}{7}, -\frac{3}{7}$ (b) $\frac{2}{7}, -\frac{6}{7}, -\frac{3}{7}$ (c) $-\frac{2}{7}, -\frac{6}{7}, \frac{3}{7}$ (d) $-\frac{2}{7}, -\frac{6}{7}, -\frac{3}{7}$

ASSERTION-REASON BASED QUESTIONS

19. In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of a.
(c) A is true but R is false
(d) A is false but R is true.

Assertion (A) : $f(x) = -2 + |x - 1|$ has minimum value at x = 1.

Reason (R) : When $\frac{d}{dx}\left(f(x)\right) < 0$ for all $x \in (a - h, a)$ and $\frac{d}{dx}\left(f(x)\right) > 0$ for all $x \in (a, a + h)$ where 'h' is an infinitesimally small positive quantity, then f(x) has a minimum at x = a. provided f(x) is continuous at x = a.

20. In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is false but R is true
(d) Both A and R are false.

Assertion (A): Let f : R $\rightarrow$ Rdefined by $f(x) = \frac{|x-1|}{x-1}$ is a bijective function.

Reason (R): A function f : A $\rightarrow$ B is said to be bijective if range of the function is codomain.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Find the principal value of $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

OR

Check whether the function f: N→N given by f (x) = x^2 is bijective or not.

22. Find the interval in which the function $y = x(x - 3)^2$ decreases.

23. A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

24. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2}\, dx$

OR

Evaluate $\int_0^1 x(1-x)^n\, dx$

25. If the product of two positive numbers is 9(sum), find the numbers so that the sum of their squares is minimum

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate: $\int \frac{(x+2)dx}{\sqrt{x^2+5x+6}}$

27. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean of the probability distribution.

OR

A and B throw a pair of dice alternatively. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$.

28. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$.

OR

Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$.

29. Find the general solution of the differential equation $(x\log x)\frac{dy}{dx} + y = \frac{2}{x}\log x$.

OR

Find the general solution of the differential equation
$ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy$, when $y \neq 0$.

30. Solve the following equation graphically

Minimum Z = 5x + 10y

Subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$

31. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ then prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Using integration, find the area of the region $\{(x, y): y^2 \leq 4x, x + y \leq 3, x, y \geq 0\}$.

33. Define a relation R in the Set N × N as follows;

For $(a, b), (c, d) \in N \times N$, (a, b) R(c, d) iff ad = bc, prove that R is an equivalence relation

OR

Consider $f: [0, \infty) \to [-5, \infty)$ given by $f(x) = 4x^2 + 4x - 5$. Prove that f is one-one and onto.

34. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

Find the image of the point (1,6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

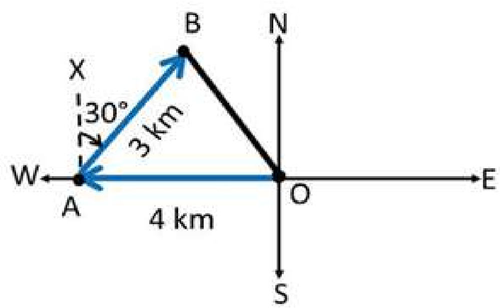
35. If $A=\begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$, find A^{-1} and hence solve the following system of equations

$2x + y + 3z = 3$,$4x - y = 3$ and $-7x + 2y + z = 2$

SECTION-E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study question have three sub-parts (i),(ii),(iii) of marks 1,1,2 respectively. The third case study question has two sub-parts of 2 marks each).

36. **Case-Study1:** Nanci walks 4km towards west, then she walk 3km in a direction 30°east of north and stops.



Based on the above information, answer the following questions

(i) Find the position vector of A

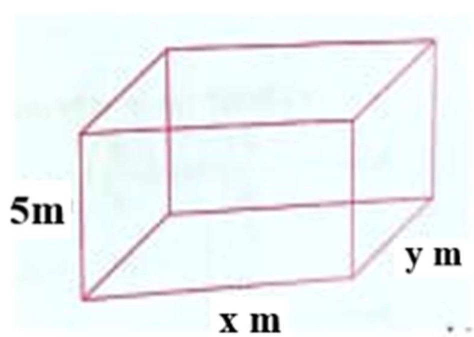
(ii) Find area of triangle OAB.

(iii) Determine Nanci displacement from her initial point of departure.

OR

Find the direction cosines of Nanci displacement from her initial point of departure.

37. **Case-Study 2**: On the request a housing society, a construction agency design a tank with the help of an architect. Tank consists of rectangular base with rectangular sides open at the top, so that its depth is 5m and volume is $20m^3$ as shown below



i) If x and y represents the length and breadth of its rectangular base, then find the relation between the variable.

ii) If construction of tank cost Rs 70 per sq. metre for the base and Rs.45 per square metre for sides, then find cost ‘C’ in terms of x.

iii) If ‘C’ is to be minimized find the value of ‘x’

OR

If $C = 80 + 80\left(x + \frac{4}{x}\right)$ and we want to minimize the cost 'C' then what will the value of 'x'.

38. **Case-Study 3**:After observing attendance register of Class-XII, Academic committee comes on conclusion that, 30% students have 100% attendance and 70% students are irregular to attend class. It was found that 80% of all students who have 100% attendance secured 95% and above in XII Board exam where 10% irregular students have secured 95% and above marks.

(i) At the end of the session, one student is chosen at random from the class has secured 95% and above marks, find the probability that the students has 100% attendance.

(ii) Find the total probability of the selected student having 95% and above marks in the class.